Internship Report: Is Gravity Quantum?

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In this report, I will be discussing the topic that I have studied during my Internship at International Center for Theoretical Physics (ICTP) under the supervision of Angelo Bassi[a] and Sandro Donadi[b]. The main topic of this paper is the recent experimental proposals to test the quantumness of gravity. I will start with some preliminaries and later discuss the proposed experiments. Then, I will discuss the objections to those experiments and possible answers to those objections.

I. INTRODUCTION

An experiment aimed to measure a quantum gravitational effect is proposed by Bose *et.al* [1] and by Marletto and Vedral [2]. The main idea of these experiments was to use a quantum informatic approach to show that an entanglement created between particles due to only gravity proves that gravity itself must be in a superposition. The details of the experiment will be discussed in section III. Later, Christodoulou and Rovelli [3] approached the same problem in the framework of general relativity and point out that this effect could be an evidence for the superposition of spacetime geometries and adress some of the objections in the literature [4]. We will also discuss these topics in the sections IV and V.

II. PRELIMINARIES

In this section, I give a brief summary of tools that are needed to grasp the quantum informatic approach to the problem. Although, not everything in this section is needed for the rest of the paper. I will mostly follow the lecture notes of Angelo Bassi on Advanced Quantum Mechanics and Lecture notes of Michael Walter and Maris Ozols on Quantum Information Theory as well as [5], [6] and [7].

Before starting to the next section, we need some definitions. First, we denote the set of all linear operators on from $\mathcal{H}_A$ to $\mathcal{H}_B$ as $\mathcal{L}(\mathcal{H}_A, \mathcal{H}_B)$ and $\mathcal{L}(\mathcal{H})$ if it is from $\mathcal{H}$ to $\mathcal{H}$. Similarly, set of all linear bounded operators are denoted $\mathcal{B}(\mathcal{H})$ and set of all trace class operators $\mathcal{T}(\mathcal{H})$. In a non-rigorous way, one can think of the bounded operators as the maps from bounded sets to bounded sets . And trace-class refers to the fact that operators has a well defined trace which is not infinite. Furthermore, if we assume a finite dimensional space, every bounded linear operator is trace-class. Hence, we can use $\mathcal{T}(\mathcal{H})$ and $\mathcal{B}(\mathcal{H})$ interchangeably since we deal with finite dimensional Hilbert Spaces.

A. Axioms of Quantum Mechanics

1. **States:** Every physical system is associated with an Hilbert Space $\mathcal{H}$ and states are represented by linear, positive and trace-1 density matrices ρ

$$\rho = \sum_k p_k \left|\psi_k\right\rangle \left\langle\psi_k\right| \tag{1}$$

   if we have the exact information of the system, i.e., $\rho = |\psi\rangle\langle\psi|$ it is called *pure state*. If it is of the form 1, it is called *statistical mixture*. And recall that we denote the set of bounded linear operators on Hilbert space as $\mathcal{B}(\mathcal{H})$, hence $\rho \in \mathcal{B}(\mathcal{H})$

2. **Evolution:** States evolve according to Schrödinger Equation

$$i\hbar \frac{d}{dt}\rho_t = [H, \rho_t]$$

3. **Observables:** The observables are represented by self-adjoint operators on $\mathcal{H}$

$$A \to \hat{A} : \hat{A}\left|a_n\right\rangle = a_n \left|a_n\right\rangle$$

   such that

$$\hat{A} = \sum_n a_n \left|a_n\right\rangle \left\langle a_n\right|$$

4. **Measurement:** The probability of obtaining the outcome a_n from the system ρ is $P[a_n]$ and it is given by

$$P[a_n] = \sum_k p_k |\left\langle a_n|\psi_k\right\rangle|^2 = \langle a_n|\rho|a_n\rangle$$

   If we define a projection operator $\mathcal{P}_n = \langle a_n|a_n\rangle$, we can also write

$$P[a_n] = \mathrm{Tr}[\mathcal{P}_n\rho]$$

   Similarly, we can write the average value of an obervable $\hat{A}$ as

$$\langle\hat{A}\rangle = \mathrm{Tr}[\hat{A}\rho]$$

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5. **Collapse:** If we obtain the outcome a_n after the measurement the state ψ collapses to $|a_n\rangle$, hence, ρ collapses to $|a_n\rangle\langle a_n| = \mathcal{P}_n$.

$$\rho \rightarrow \frac{\mathcal{P}_n \rho \mathcal{P}_n}{\mathrm{Tr}[\mathcal{P}_n \rho]}$$

where the division by trace is to make sure that trace is equal to one. This measurement type is called *selective*. One can also define a *non-selective measurement* by imposing that more than one outcome is possible (like a measurement yields a superposition of states). In that case one has

$$\rho \rightarrow \sum_n \mathrm{Tr}[\mathcal{P}_n \rho] \frac{\mathcal{P}_n \rho \mathcal{P}_n}{\mathrm{Tr}[\mathcal{P}_n \rho]} = \mathcal{P}_n \rho \mathcal{P}_n$$

Therefore, non-selective measurements can turn pure states into statistical mixture.

B. Entanglement

Consider a bipartite system, i.e., total Hilbert space $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$. If we can not express the state of the total system as the linear combination of the states of the subsystems, i.e.,

$$\rho \neq \sum_{ij} \lambda_{ij} \rho_i^{(1)} \otimes \rho_j^{(2)}$$

it is called **entangled state**. If we can express ρ as above, with the conditions that λ_{ij}'s are real positive numbers and $\sum_{ij} \lambda_{ij} = 1$ as well as $\rho_i^{(1)}$'s and $\rho_j^{(2)}$'s are good density matrices, i.e., positive,linear,trace-1.

C. Quantum Operations

A quantum operation is a linear, completely positive(CP) trace nonincreasing map, i.e., quantum operations has the general form

$$\rho \rightarrow \Lambda(\rho)/\mathrm{Tr}(\Lambda(\rho)) \tag{2}$$

such that $\mathrm{Tr}(\Lambda(\rho)) \leq 1$ for any state ρ. This can also be written as

$$\Lambda(\rho) = \sum_i V_i \rho V_i^\dagger \tag{3}$$

with $\sum_i V_i^\dagger V_i \leq 1$. This operation takes place with the probability $\mathrm{Tr}(\Lambda(\rho))$. If we have the special case where this probability is equal to 1, i.e., if we have $\sum_i V_i^\dagger V_i = 1$, it is called *quantum channel* or *deterministic operation* and the operators V_i's are called *Kraus Operators*. Most of the time, Quantum Operation and Quantum Channel is used interchangeably because we assume this condition to hold.

D. Local Operations and Classical Channel (LOCC)

Local Operations and Classical Channel, shortly stated as LOCC, is a paradigm in which distant parties (call Alice and Bob) are only allowed to perform Local Operations and send each other classical information. To be able to have a more concrete understanding, we define subclasses of Quantum Operations:

1. **C1 - Local Operations (LO)**: In this class of operations, there is no communication between Alice and Bob. Hence, they have the form

$$\Lambda_{AB} = \Lambda_A \otimes \Lambda_B \tag{4}$$

where $\rho \in \mathcal{B}(\mathcal{H}_A \otimes \mathcal{H}_B)$.

2. **C2 - one-way LOCC operations:** In this class, the information is allowed only one-way. For simplicity let's assume that it is allowed from Alice to Bob. Then, Alice can perform any LO and send a classical information to Bob and he can also do a LO depending or not depending on the information. But the opposite is not true. A one-way LOCC operation $\Lambda_{AB} : \mathcal{B}(\mathcal{H}^A \otimes \mathcal{H}^B) \rightarrow \mathcal{B}(\mathcal{K}^A \otimes \mathcal{K}^B)$ can be defined as

$$\Lambda_{AB}(\rho) = \sum_{ij} (I_2^A \otimes W_{ij}^B)(V_i^A \otimes I_1^B)\rho(V_i^{A\dagger} \otimes I_1^B)(I_2^A \otimes W_{ij}^{B\dagger})$$

where I_1^A, I_1^B and I_2^A are the identity operators on hilbert spaces $\mathcal{H}^A$, $\mathcal{H}^B$ and $\mathcal{K}^A$ respectively. We also demand the condition that $\sum V_i^{A\dagger} V_i^A = I_A$ and $\sum W_{ij}^{B\dagger} W_{ij}^B = I_B$ hold. The j in the double sum refers to the fact that the operations made by Bob may depend on the outcome of the operations of Alice but the opposite is not true.

3. **C3 - two-way LOCC operations:** This class of operators are the generalized version of the previous one. Therefore, it gets complicated easily with increasing number of indices. In this class of operators, both Alice and Bob are allowed communicate, therefore both of their operations may or may not depend on the all the previous outcomes of the operations. A two-way LOCC operation $\Lambda : \mathcal{B}(\mathcal{H}_1^A \otimes \mathcal{H}_1^B) \rightarrow \mathcal{B}(\mathcal{H}_{n+1}^A \otimes \mathcal{H}_{n+1}^B)$ can be defined with the list of Hilbert spaces $H_1^A, \cdots, H_{n+1}^A$ and $H_1^B, \cdots, H_{n+1}^B$ as

$$\Lambda(\rho) = \sum_{i_1 \dots i_{2n}} N_{i_1 \dots i_{2n}}^{AB} \rho N_{i_1 \dots i_{2n}}^{AB\dagger} \tag{5}$$

where $\rho \in \mathcal{B}(\mathcal{H}_1^A \otimes \mathcal{H}_1^B)$ and $V_{i_1 \dots i_{2n}}^{AB}$ is given by

$$N_{i_1 \dots i_{2n}}^{AB} := (I_{n+1}^A \otimes W_{2n}^{i_{2n},\dots,i_1})(V_{2n-1}^{i_{2n-1},\dots,i_1} \otimes I_n^B) \cdots (I_2^A \otimes W_2^{i_2,i_1})(V_1^{i_1} \otimes I_1^B) \tag{6}$$

with the family of operators

$$\left(V_{2k+1}^{i_{2k+1},\ldots,i_1} : \mathcal{H}_k^A \to \mathcal{H}_{k+1}^A\right)$$
$$\left(W_{2l}^{i_{2l},\ldots,i_1} : \mathcal{H}_l^B \to \mathcal{H}_{l+1}^B\right)$$

such that for each $k = 0, \cdots, n-1$ and $l = 0, \cdots, n$, we have

$$\sum_{i2k+1} = (V_{2k+1}^{i_{2k+1},\ldots,i_1})(V_{2k+1}^{i_{2k+1},\ldots,i_1\dagger}) = I_{k+1}^A$$
$$\sum_{i2l+1} = (W_{2l}^{i_{2l},\ldots,i_1})(W_{2l}^{i_{2l},\ldots,i_1\dagger}) = I_l^B$$

where I_k^A and I_l^B denotes the identity operators on $\mathcal{H}_k^A$ and $\mathcal{H}_l^B k$ respectively.

4. **C4 - Separable Operations:** The definition for the two-way LOCC operations were quite complicated, luckily, however, we have another class of operations called *Separable Operations* which are more general then LOCC operations. A separable operation is a map $\Lambda : \mathcal{B}(\mathcal{H}_1^A \otimes \mathcal{H}_1^B) \to \mathcal{B}(\mathcal{H}_2^A \otimes \mathcal{H}_2^B)$

$$\Lambda(\rho) = \sum_i (V_i \otimes W_i)\rho(V_i \otimes W_i)^\dagger \tag{7}$$

with $\sum_i (V_i \otimes W_i)(V_i \otimes W_i)^\dagger = I_1^A \otimes I_1^B$ where I_1^A and I_1^B are the identity operators in $\mathcal{H}_1^A$ and $\mathcal{H}_1^B$ respectively.

There are also another class of operations called *PPT operations* which we will not discuss here. And it is important to note the inclusion between classes as $C1 \subset C2 \subset C3 \subset C4$. In other words, any LOCC operation can be put into the form of separable operations.

E. LOCC Constraint

LOCC constraint refers to the fact that LOCC operations cannot increase entanglement. In order to show this mathematically we need some tools. Rest of the this section will be some definitions, lemmas which will allow us the prove LOCC constraint. We will closely follow the (CITE)

Defintion 1 (Vectorization)**.** *Let $M \in \mathcal{L}(\mathcal{H}_A \mathcal{H}_B)$ where $\mathcal{H}_A = \mathbb{C}^\alpha$ with the orthonormal basis $|a\rangle$ and $\mathcal{H}_B = \mathbb{C}^\beta$ with the orthonormal basis $|b\rangle$. Then,* ***vectorization*** *of M is given by*

$$|M_{AB}\rangle := \sum_{a,b} \langle b|M||a\rangle \left(|a\rangle \otimes |b\rangle\right) \in \mathcal{H}_A \otimes \mathcal{H}_B \tag{8}$$

This definition can easily be visualized as taking every column of a matrix and stacking them to create one large column vector.

A very useful vectorization identity follows from the definition is the following:

$$(A \otimes B)\,|M\rangle = \left|BMA^T\right\rangle \tag{9}$$

We already defined the entangled states and the separable states. But we also need a measure of entanglement to conclude how much entangled a state is. To to that first we define the Schmidt Rank.

Defintion 2 (Schmidt Rank)**.** *Any $|\psi_{AB}\rangle$ can be written as*

$$|\psi_{AB}\rangle = \sum_i^r s_i\,|e_i\rangle \otimes |f_i\rangle \tag{10}$$

where the $s_i > 0$, $\{|e_i\rangle\} \subset \mathcal{H}_A$ and $\{|f_i\rangle\} \subset \mathcal{H}_B$. This composition is called Schmidt decomposition, and r is called Schmidt rank.

Defintion 3 (Entanglement Rank:)**.** *Let ρ_{AB} be a bipartite state. Then we write $\rho_{AB} \in \mathrm{Ent}_r(\mathcal{H}_A : \mathcal{H}_B) \subset \mathcal{B}(\mathcal{H}_A \otimes \mathcal{H}_B)$ if*

$$\rho_{AB} = \sum_i |\psi_i\rangle\langle\psi_i| \tag{11}$$

where each $|\psi_i\rangle \in (\mathcal{H}_A \otimes \mathcal{H}_B)$ has Schimdt rank at most r. Then, the entanglement rank of ρ_{AB} is said to be the r, i.e., $\rho_{AB} \in \mathrm{Ent}_r(\mathcal{H}_A : \mathcal{H}_B)$.

Note that the entanglement rank of a separable state is 1 and gets larger as it gets more entangled. It is a rough way of measuring the entanglement since it can only take integer values but we will stick with definition and show that, the entanglement doesn't increase with Separable operations as we defined in subsection II D.

Theorem 1 (Separable Map constraint)**.** *Let $\Lambda : \mathcal{B}(\mathcal{H}_A \otimes \mathcal{H}_B) \to \mathcal{B}(\mathcal{H}_\mathcal{C} \otimes \mathcal{H}_\mathcal{D})$ be a separable map, and let there be a separable state with entanglement rank r, i.e., $\rho \in \mathrm{Ent}(\mathcal{H}_A : \mathcal{H}_B)$. Then entanglement rank of $\Lambda(\rho)$ is also r , i.e., $\Lambda(\rho) \in \mathrm{Ent}(\mathcal{H}_C : \mathcal{H}_D)$*

Proof. Recall from previous section that we can write the action of separable map on ρ as

$$\begin{aligned}\Lambda(\rho) &= \sum_i (V_i \otimes W_i)\rho(V_i \otimes W_i)^\dagger \\ &= \sum_{ij} (V_i \otimes W_i)(|\psi_j\rangle\langle\psi_j|)(V_i \otimes W_i)^\dagger \end{aligned}\tag{12}$$

where we also plugged $\rho = \sum_j |\psi_j\rangle\langle\psi_j|$. Now we use the identity Eq 9 to write

$$\Lambda(\rho) = \sum_{ij} \left|V_i \psi_j W_i^T\right\rangle \left\langle V_i \psi_j W_i^T\right| \tag{13}$$

Now, since Schmidt rank of$|\psi_i\rangle$ is smaller than r, and Schmidt rank of $\left|V_i \psi_j W_i^T\right\rangle$ is smaller than the Schmidt rank of $|\psi_i\rangle$ we conclude that Entanglement rank of $\Lambda(\rho)$ is r. $\square$

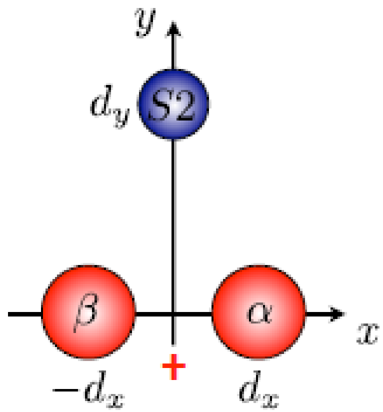

FIG. 1. Experimental Setup from [9]

And it follows directly from the theorem the following corollary.

Corollary 1.1 (LOCC Constraint)**.** *Separable states remains separable under the action of separable operations. Also, from the inclusion $C3 \subset C4$, we can also say the same for LOCC,i.e., LOCC operations leaves the separable operations separable.* □

III. THE EXPERIMENTS

In this section we will investigate the proposed experiments to test the Quantum Nature of Gravity.

A. Early Proposals

One of the early proposal is dating back to 1945 Chapel Hill Conference ([8]). Feynman proposed the idea to prepare a mass in a superposition o two different location and let it interact with the gravitational field. Since, both of the branches will evolve differently due to their position, Feynman concluded that it can be seen some quantum interference effect with some experiments.

Later, a very similar idea came from Bahrami *et.al* [9]. In it, they proposed a very similar experiment as Feynman did. In this experiment, we again have a mass in a superposition of two different location. However, this time instead of putting them in a gravitational field of massive object, we use another test particle as in figure 1.

It is important to note that these experiments are not the same. More about these experiments will be discussed in section VI.

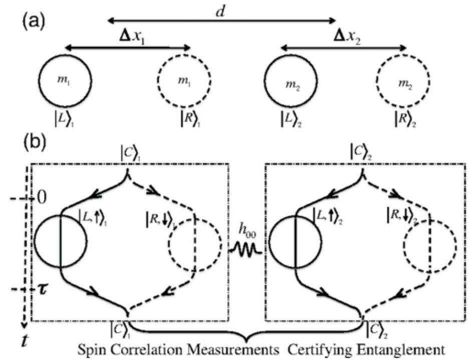

FIG. 2. Experiment setup from *Bose et.al* [1]

B. Proposed Experiment in [1],[2] (BMV effect)

The experiment relies on the fact that 2 masses m_1 and m_2 each is in a superposition of two different locations, namely left and right, interact via only gravity should create and entanglement.

In other words, consider two different masses each is in a superposition of left and right are separated by a distance d as in the figure 2. Then the initial state can be written as

$$\begin{aligned} |\Psi(t=0)\rangle_{12} =& \frac{|L_1\rangle + |R_1\rangle}{\sqrt{2}} \otimes \frac{|L_2\rangle + |R_2\rangle}{\sqrt{2}} \\ =& \frac{1}{2}\big(|L_1 L_2\rangle + |L_1 R_2\rangle + |R_1 L_2\rangle + |R_1 R_2\rangle \big) \end{aligned} \tag{14}$$

where we used a shorthand $|L_1\rangle \otimes |L_2\rangle \equiv |L_1 L_2\rangle$. Also from now on let's assume that $m_1 = m_2 = m$ Then, after some time τ the state will evolve as

$$\begin{aligned} |\Psi(t=\tau)\rangle = \frac{1}{2}\Bigg(& |L_1 L_2\rangle e^{i\phi_1} + |L_1 R_2\rangle e^{i\phi_2} \\ & + |R_1 L_2\rangle e^{i\phi_3} + |R_1 R_2\rangle e^{i\phi_1} \Bigg) \end{aligned} \tag{15}$$

where we defined

$$\phi_1 = \frac{Gm^2\tau}{\hbar d} \phi_2 = \frac{Gm^2\tau}{\hbar(d+\Delta x)} \quad \phi_3 = \frac{Gm^2\tau}{\hbar(d-\Delta x)} \tag{16}$$

Factoring out the common phase and rearranging, we get

$$\begin{aligned} |\Psi(t=\tau)\rangle = \frac{1}{2}e^{i\phi_1}\Bigg[& |L_1\rangle \Big(\underbrace{|L_2\rangle + e^{i\Delta\phi_{21}} |R_2\rangle}_{|\alpha\rangle} \Big) \\ & + |R_1\rangle \Big(\underbrace{e^{i\Delta\phi_{31}} |L_2\rangle + |R_2\rangle}_{|\beta\rangle} \Big) \Bigg] \end{aligned} \tag{17}$$

Where we defined $\Delta\phi_{ij} \equiv \phi_i - \phi_j$. Now, we conclude that, as long as the sates $|\alpha\rangle$ and $|\beta\rangle$ are not the same, i.e., $\Delta\phi_{21} + \Delta\phi_{31} = 2n\pi$, the state cannot be factorized. Therefore, it is an entangled state. The process of creating an entangled state via gravitation is started to being called as Bose-Marletto-Vedral (BMV) effect in the literature.

Since we showed that the state become entangled via gravitational interaction, we now turn our attention to the quantum information side. In [1], authors claim that there is 2 main assumption underlying their reasoning. 1) They assume locality and causality such that gravity is mediated by a field and not an action at a distance and 2) a theorem from quantum information called Local Operation and Classical Communication (LOCC) Constraint is correct. The theorem states that, one cannot increase the entanglement of states by using LOCC operations, and therefore cannot create entanglement from pure states. (for a detailed discussion see section II E). After accepting this two assumption, the reasoning goes as follows: Any external field including other masses around them can only make Local Operations (LO) on the masses while gravity can only create a classical channel (CC) between them. Hence, by theorem, there should be not a entanglement between masses. In the case of seeing an entanglement, one concludes that *"gravity is a CC"* is a wrong statement, therefore, gravity must be a quantum channel, i.e., gravity must be quantum mechanical in nature.

IV. A RELATIVISTIC APPROACH TO BMV

A General Relativistic approach to the so called BMV effect came from Christodoulou and Rovelli [3]. From this point of view, they claim that this effect can be counted as a proof of *quantum superposition of different spacetime geometries*. And then, they discuss the current objections to this effect in the current literature which we will be discussing in detail in section V.

A. On the Meaning of Planck Mass

Recall that we calculated the phase different of different branches in the section III and found as in equation 16. Now, we can equivalently write the phase difference as

$$\delta\phi = \frac{Gm^2\tau}{\hbar d_i} = \alpha\left(\frac{m}{m_{planck}}\right)^2 \tag{18}$$

where d_i is the distances as in equation 16, $\alpha = \frac{c\tau}{d_i}$ and $m_{planck} = \sqrt{\hbar c/G}$. Looking at the equation 18, They conclude that planck mass is the scale at which quantum superposition of gravity (or spacetime geometries) becomes detectable. They say that m/m_{planck} determines the physical effect and α is a large multiplicative factor making it measurable. We discuss more on this in the discussion part.

B. General Covariant Treatment of the BMV effect

The main idea is that we can treat the system as static in the limit because the time τ required is much greater than the light travel time d/c between masses which is the time where the system is not static. Also we can neglect the displacement of particles. Therefore, we can just focus on the static phase and use the Minkowski background with a perturbation.

Now, consider a the same experimental setup with 2 spherical masses apart from each other at a distance d and their radius $R \ll d$. Then, the metric becomes

$$ds^2 = (1 + 2\Phi/c^2)dt^2 - d\vec{x}^2 \tag{19}$$

where Φ is the sum of the newtonian potentials for two particles. And we take newtonian potential inside each particle to be constant. Therefore, for each particle, metric inside the particles approximately becomes (since $R \ll d$)

$$ds^2 = \left(1 - \frac{2Gm}{Rc^2} - \frac{2GM}{dc^2}\right)dt^2 - d\vec{x}^2 \tag{20}$$

from this we calculate the proper time

$$\begin{aligned} s = \int_0^t ds &= \int_0^t \sqrt{1 - \frac{2Gm}{Rc^2} - \frac{2GM}{dc^2}}dt \\ &\sim t\left(1 - \frac{Gm}{Rc^2} - \frac{GM}{dc^2}\right) \end{aligned} \tag{21}$$

Since radius is much greater than the Schwarzschild radius, i.e., $R \gg r_m = 2Gm/c^2$, the last term is small.

Now, let's keep this in mind and have a look at the same experimental setup but this time rewrite the state as

$$|\Psi\rangle = \frac{1}{2}\bigg(|L_1L_2\rangle + |L_1R_2\rangle + |R_1L_2\rangle + |R_1R_2\rangle\bigg) \otimes |g\rangle \tag{22}$$

where we also added $|g\rangle$ as the quantum state of gravity. Here, we don't say anything about state of the gravity other than the fact that it can be in a superposition. This is the key idea for BMV effect to take place.

It is also important to point out that the metric defined before in 19 with different values of d's is not diffeomorphic to each other. In other words, the different between spacetime metrics in different branches is not a gauge difference. This is an important fact that will be discussed in the discussion section.

Metric in different branches is different depending on the distance d. We will denote the metric in each branch as $g_{LL}, g_{RR}, g_{LR}, g_{RL}$ depending on which branch it is in. Hence,

$$|\Psi\rangle = \frac{1}{2}\bigg(|L_1L_2g_{LL}\rangle + |L_1R_2g_{LR}\rangle \tag{23}$$

$$+ |R_1L_2g_{RL}\rangle + |R_1R_2g_{RR}\rangle\bigg) \tag{24}$$

Now, assume that the distance d is big in three of the four branches and small in one of them (RL) so that the last term in equation 21 becomes important only in that one branch. Therefore, all other branches will experience almost the same proper time while the RL branch there will be a difference equal to the last term. Therefore time evolution of the system will result in this branch to evolve different than the others with the phase

$$\delta\phi = -\frac{mc^2 \delta s}{\hbar} = \frac{Gm^2 \tau}{\hbar d} \tag{25}$$

which is exactly the same one found in [1]. And if we bring back the particles together again, the state will be

$$|\Psi\rangle = \frac{1}{2}\bigg(|L_1 L_2\rangle + |L_1 R_2\rangle + e^{i\delta\phi} |R_1 L_2\rangle + |R_1 R_2\rangle \bigg) \otimes |g\rangle \tag{26}$$

And this state is an entangled state. There it is concluded in the paper that if we detect BMV effect then it is a strong evidence in favor of, the existence of spacetime superposition.

V. OBEJCTIONS

There has been some objections in the literature, namely from Anastopoulos and Hu ([4]) stating that in the weak field limit in which these experiments function, the interaction is determined by scalar constraint of General Relativity(GR) and not by the dynamical equations. Therefore they claim that the relevant dof's in the experiment are pure gauge and the outcome of the experiment won't tell us about quantum nature of gravity.

In other words, They say that Newtonian Potential is pure gauge and the transverse-traceless perturbation are the true dynamical dof of gravity. And since the experiment does not probe this dof, it cannot test quantumness of gravity. If one wants to show the quantum nature of gravity, the only way is to detect gravitons.

Spin zero sector of gravitational perturbations are related to matter density, spin 1 sector is is related to vorticity of matter and spin 2 sector describes the gravitational waves. Hence only the spin 2 component carries the true dof. Therefore, they object the claims of the authors of [1] and [2].

Although, the debate seems to be settled down and the papers ([1] and [2]) are published with 2 referees being in favor against 1, there are still some questions remained to be answered. More about these objections will be discussed in section VI.

VI. DISCUSSION

In this chapter, I will briefly talk my opinion about the everything written in this paper so far.

I think It is very important to understand what we mean by *quantumness* or *quantum nature of* gravity. Many of the papers in the literature avoid to give such a definition but as far as I understand from my readings, there is a consensus that the definition for quantumness is *to be able to be in a superposition.* I believe, for now, this is the best definition.

Now, starting from the previous proposals, in [2], authors claims that the proposal of Feynman in 1945 [8] is not enough to conclude that gravity is quantum in nature. I believe this is true, because having a mass in a superposition of two different location and putting it in a gravitational field can only induce phase shift between different branches. And this does not require a quantum field. A classical gravitational field can also give the same results. However, it is important to note that, as I also mentioned in section III A, Feynman's proposal is not the same as in the [9]. The former only talks about the evolution of a single mass which is in a superposition, while the latter talks about evolving two different masses. In other words, we allow both particle to evolve in [9]. Therefore, they become entangled, and if we make measurement after some time, we can conclude whether the gravity was in a superposition or not as the authors claim. In this sense, it is no different from the [1] and [2].

I also would like to write about the objections to these experiments in the following days. But I think it requires some time to fully understand who is correct. It seems like both the objections and the answers to those objections are reasonable. For now, I will leave this part like this and hopefully return with convincing arguments. I have also heard about the the paper [10] lately. I think it will also have some help to settle down this debate, at least for me.

ACKNOWLEDGMENTS

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